Volatility modeling: state of the art & outlook

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Outline

- The vanilla trader
- The exotic trader
- The local volatility model
- Stochastic volatility models
- Local-stochastic volatility models
- Beyond models?

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Two ideal types of derivative traders: the vanilla trader

- Risk-manages vanillas
- Hedge instruments: underlyings/futures/forwards
- Pricing function $P(t, S, \bullet)$. $\bullet \equiv$ model parameters
- P&L of delta-hedged position – assume has sold option + assume zero interest rate

$$P&L = -\left( P(t + \delta t, S + \delta S, \bullet) - P(t, S, \bullet) \right) + \Delta \delta S$$

- Use Black-Scholes pricing function and take $\Delta = \frac{dP}{dS}$

$$\frac{dP}{dt} + \frac{1}{2} \sigma^2 S^2 \frac{d^2 P}{dS^2} = 0$$

- Expand at order 1 in $t$, order 2 in $\delta S$. P&L reads:

$$P&L = \frac{1}{2} S^2 \frac{d^2 P}{dS^2} \left( \frac{\delta S^2}{S^2} - \sigma^2 \delta t \right)$$

- Notice there are additional contributions to the P&L:

$$P&L = \frac{1}{2} S^2 \frac{d^2 P}{dS^2} \left( \frac{\delta S^2}{S^2} - \sigma^2 \delta t \right) + \frac{dP}{d\hat{\sigma}} \delta \hat{\sigma} + \frac{d^2 P}{dS d\hat{\sigma}} \delta S \delta \hat{\sigma} + \frac{1}{2} \frac{d^2 P}{d\hat{\sigma}^2} \delta \hat{\sigma}^2 + \cdots$$

- First portion (Gamma/Theta P&L) is by no means small
Two ideal types of derivative traders: the exotic trader

- Risk-manages exotics
- Uses vanillas/delta-hedged vanillas as hedge instruments
- Pricing function takes as input vanilla option prices / vanilla implied volatilities:

\[ P(t, S, \hat{\sigma}_{KT}, \bullet) \quad P(t, S, O_{KT}, \bullet) \]

- Denote by \( A_i \) prices of hedging instruments: \( S \) and \( O_{KT} \)
- P&L of delta and vega-hedged option position – assume zero interest rate

\[
P\&L = -\left( P(t + \delta t, A + \delta A, \bullet) - P(t, A, \bullet) \right) + \sum_i \frac{dP}{dA_i} \delta A_i
\]

- Expanding at order 1 in \( \delta t \), order 2 in \( \delta A \):

\[
P\&L = -\frac{dP}{dt} \delta t - \frac{1}{2} \sum_{ij} \frac{d^2P}{dA_i dA_j} \delta A_i \delta A_j
\]

\( \Rightarrow \) How to tell if \( P(t, A, \bullet) \) is a non-nonsensical pricing function?

- There exists a positive matrix \( \hat{C} \) such that \( \frac{dP}{dt} = \frac{1}{2} \sum_{ij} A_i A_j \frac{d^2P}{dA_i dA_j} \hat{C}_{ij} \)
- P&L reads:

\[
P\&L = -\frac{1}{2} \sum A_i A_j \frac{d^2P}{dA_i dA_j} \left( \frac{\delta A_i}{A_i} \frac{\delta A_j}{A_j} - \hat{C}_{ij} \delta t \right)
\]
Two *ideal types* of derivative traders: *the exotic trader – 2*

- The $\hat{C}_{ij}$ are called break-even covariances
- For a given model, they are functions of $(t, A_i, \bullet)$ with $\bullet \equiv$ model parameters
- They can equivalently be expressed in terms of volatilities and correlations
- In what follows, use implied volatilities $\hat{\sigma}_{KT}$ rather than vanilla option prices
- Ideally we would like to be able to specify breakeven levels:
  - Volatilities of $\hat{\sigma}_{KT}$
  - Correlations of the $\hat{\sigma}_{KT}$ among themselves and with $S$
- Let us now assess different types of models
  - The Local Volatility (LV) model
  - Stochastic volatility (SV) models
  - Local-stochastic volatility (LSV) models
- But wait – do we actually need a model?
The LV model

- Historically, LV model introduced as extension of BS with deterministic inst. vol.

\[ dS_t = (r - q)S_t dt + \sigma(t, S_t)S_t dW_t \]

- \( \sigma(t, S) \) backed out of call prices \( O_{KT} \) by the Dupire formula

\[ \sigma(T, K)^2 = 2 \frac{dO}{dT} + qO + (r - q)K \frac{dO}{dK} \]

- LV function is not a fundamental object, only has an ancillary function

- Pricing function in LV model – that trader uses – is:

\[ P(t, S, \hat{\sigma}_{KT}) \text{ or } P(t, S, O_{KT}) \]

\( \Rightarrow \) No parameter ??

- In LV model for a fixed LV function all assets are functions of \( t, S \)

\( \Rightarrow \) 1-dimensional Markov representation in terms of \( t, S \), for all assets

- With a fixed LV function, pricing function \( P^{LV}(t, S, \sigma) \)

\[ \hat{\sigma}_{KT} \equiv \Sigma^{LV}_{KT}(t, S, \sigma) \]
The LV model – 2

Full P&L of naked exotic option position reads - now with interest rates:

\[ P\&L = - \frac{dP}{dS} (\delta S - rS \delta t) - \frac{dP}{d\hat{\sigma}_{KT}} \ast (\delta \hat{\sigma}_{KT} - \mu_{KT} \delta t) \]

\[ - \frac{1}{2} S^2 \frac{d^2 P}{dS^2} \left[ \frac{\delta S^2}{S^2} - \sigma^2(t, S) \delta t \right] \]

\[ - \frac{d^2 P}{dS d\hat{\sigma}_{KT}} \ast S \hat{\sigma}_{KT} \left[ \frac{\delta S}{S} \frac{\delta \hat{\sigma}_{KT}}{\hat{\sigma}_{KT}} - \sigma(t, S) \nu_{KT} \delta t \right] \]

\[ - \frac{1}{2} \frac{d^2 P}{d\hat{\sigma}_{KT} d\hat{\sigma}_{K'T'}} \ast \hat{\sigma}_{KT} \hat{\sigma}_{K'T'} \left[ \frac{\delta \hat{\sigma}_{KT}}{\hat{\sigma}_{KT}} \frac{\delta \hat{\sigma}_{K'T'}}{\hat{\sigma}_{K'T'}} - \nu_{KT} \nu_{K'T'} \delta t \right] \]

where the (lognormal) vol \( \nu_{KT} \) of \( \hat{\sigma}_{KT} \) is given by:

\[ \nu_{KT} = \frac{1}{\Sigma_{KT}^{LV}} \frac{d\Sigma_{KT}^{LV}}{dS} S \sigma(t, S) \]

and \( \mu_{KT} \) is given by:

\[ \mu_{KT} = \frac{d\Sigma_{KT}^{LV}}{dt} + \frac{1}{2} \sigma^2(t, S) S^2 \frac{d^2 \Sigma_{KT}^{LV}}{dS^2} + rS \frac{d\Sigma_{KT}^{LV}}{dS} \]

For a perfectly delta-hedged/vega-hedged position

\[ \Rightarrow \] The first 2 contributions vanish

\[ \Rightarrow \] Remaining P&L is of the non-nonsensical type in slide 3
The LV model – 3

- Pricing function takes as input $t$, the prices of hedge instruments – and that’s it
- Not possible to choose the vol of $\hat{\sigma}_{KT}$
  - $\nu_{KT}$ is determined by derivative of $\Sigma_{KT}^{LV}$ with respect to $S$
  - $\Sigma_{KT}^{LV}$ determined by local vol function $\sigma(t, S)$
  - Set by the market smile used for calibration

Correlations between implied vols = $\pm 100\%$

Correlations between $S$ and $\hat{\sigma}_{KT}$ = $\pm 100\%$

- Can we size up $\nu_{KT}$?
The LV model – 4

Let us focus on the vol of the ATM vol \( \hat{\sigma}_{K=S,T} \), and consider the vol \( \tilde{\nu}_{K=S,T} \). It is the volatility of the floating ATM volatility:

\[
\tilde{\nu}_{K=S,T} = \frac{1}{\Sigma_{S,T}^{LV}} \frac{d\Sigma_{K=S,T}^{LV}}{dS} S \sigma(t, S) = \frac{1}{\Sigma_{S,T}^{LV}} \frac{d\Sigma_{K=S,T}^{LV}}{d \ln S} \sigma(t, S)
\]

For a flat smile, \( \sigma(t, S) \) does not depend on \( S \) \( \Rightarrow \) neither does \( \Sigma_{K,T}^{LV} \)

\[
\Rightarrow \tilde{\nu}_{K=S,T} = 0
\]

Denote by \( S_{\tau} \) the ATM skew for maturity \( \tau \):

\[
S_{\tau} = \left. \frac{d\hat{\sigma}_{K,\tau}}{d \ln K} \right|_{K=S}
\]

At order 1 in \( S \):

\[
\frac{d\Sigma_{ST}^{LV}}{d \ln S} = S_{\tau} + \frac{1}{T} \int_{0}^{T} S_{\tau} d\tau
\]
The LV model – 5

Example of index smile. Right: \((\hat{\sigma}_{0.95S, T} - \hat{\sigma}_{1.05S, T})\) as a function of \(T\) (years)

In the LV mode the pricing function of an exotic option reads:

\[ P(t, S, \hat{\sigma}_{KT}) \quad \text{or} \quad P(t, S, O_{KT}) \]

- No additional parameter to input
- Vols of vols, correlations, are set by market smile
  - Like them ⊳ use model
  - Don’t like them ⊳ don’t use model
SV models - 1

- We would like to have a handle on the dynamics of implied vols
  - Not just let model decide – as in the LV model

- Can we model the dynamics of vanilla implied volatilities directly?

\[
\begin{align*}
  dS_t &= rS_t dt + \sigma S_t dW^S_t \\
  d\hat{\sigma}^KT_t &= \ast dt + \bullet dW^{KT}_t
\end{align*}
\]

⇒ Short answer: no
⇒ Implied volatilities of vanilla options are too complicated objects

- Other (convex) European payoffs with simpler implied volatilities?

- Yes: the logswap

\[f(S) = -2 \ln S\]

- In BS, zero interest rate for simplicity, price is given by:

\[P_T(t, S) = -2 \ln S + \frac{1}{2} \hat{\sigma}^2_T (T - t)\]

⇒ Curve of the $\hat{\sigma}_T$
SV models - 2

- Consider an increasing sequence of dates $T_i$
- Building block:
  - $+1$ logswap of maturity $T_{i+1}$
  - $-1$ logswap of maturity $T_i$

  $\Rightarrow$ payoff $= \ln \left( \frac{S_{T_{i+1}}}{S_{T_i}} \right)$

- Price in BS:

$$P_{i,i+1}(t, S) = \frac{1}{2} \hat{\sigma}_{i,i+1}^2 (T_{i+1} - T_i)$$

$$= P_{T_{i+1}}(t, S) - P_{T_i}(t, S)$$

- $\hat{\sigma}_{i,i+1}^2$ is proportional to $P$, a price $\Rightarrow$ $\hat{\sigma}_{i,i+1}^2$ has no drift
- Also holds with non-zero interest rates

- Connection with the $\hat{\sigma}_T$:

$$\hat{\sigma}_{i,i+1}^2 = \frac{\hat{\sigma}_{T_{i+1}}^2 (T_{i+1} - t) - \hat{\sigma}_{T_i}^2 (T_i - t)}{T_{i+1} - T_i}$$

- Replace finite difference by derivative $\Rightarrow$ continuous forward variances $\xi^T$

$$\xi^T = \frac{d}{dT} \left( \hat{\sigma}_T^2 (T - t) \right) \bigg|_{T=T}$$
**SV models - 3**

- General form of a stochastic volatility model:

\[
\begin{aligned}
    dS_t &= rS_t dt + \sqrt{\xi_t} dW_t^S \\
    d\xi_t^T &= \bullet dW_t^T
\end{aligned}
\]

with correlations \( \rho_{ST} dt, \rho_{TT} \).

- This is called a forward-variance model.

- All models written on the inst. variance \( V_t \) can be written this way.

- Example: Heston model – no interest rates:

\[
\begin{aligned}
    dS_t &= \sqrt{\xi_t} S_t dW_t \\
    dV_t &= -k(V_t - V^0) dt + \nu \sqrt{\xi_t} dZ_t
\end{aligned}
\]

\[
\Rightarrow
\begin{aligned}
    dS_t &= \sqrt{\xi_t} S_t dW_t \\
    d\xi_t^T &= \nu e^{-k(T-t)} \sqrt{\xi_t} dZ_t
\end{aligned}
\]

- Are the \( \xi^T \) easier to model than the \( \hat{\sigma}_{KT} \)? Yes:
  - they are driftless
  - just ensure \( \xi^T \geq 0 \)

- \( \xi^T \): 1-dimensional set of instruments

\( \Rightarrow \) Can calibrate at most one implied vol per maturity

- What should we choose for \( \bullet \)?
SV models - 4

- Empirically, ATM vols & logswap implied vols are \( \approx \) lognormal
- Typically, the inst. (lognormal) volatility of \( \hat{\sigma}_T \) has a power-law dependence on \( T \)

\[
\text{vol}(\hat{\sigma}_T) = \nu_0 \left( \frac{\tau_0}{T} \right)^\alpha
\]

- Typical realized values – for indices: \( \nu_{3m} \approx 60\%, \ \alpha \in [0.3, 0.6] \)
- Similar levels for volatilities of ATM volatilities

- How do achieve this in a forward variance model?

-dess Receipe

- Use same Brownian motion for all of the \( \xi^T \)
- Make vol of \( \xi^T \) proportional to \( (T - t)^{-\alpha} \)

\[
d\xi^T_t = \frac{\lambda}{(T - t)^\alpha} dZ_t
\]
SV models - 5

- Let’s check. By definition of $\xi^T_t$:

$$\hat{\sigma}^2_T = \frac{1}{T-t} \int_t^T \xi^T_t d\tau$$

- We have:

$$d(\hat{\sigma}^2_T) = \bullet dt + \frac{1}{T-t} \int_t^T d\xi^T_t d\tau$$

$$= \bullet dt + \frac{1}{T-t} \int_t^T \lambda \frac{1}{(\tau-t)^\alpha} dZ_t \ d\tau$$

$$= \bullet dt + \frac{\lambda}{1-\alpha} \frac{1}{(T-t)^\alpha} dZ_t$$

- Instantaneous vol of $\hat{\sigma}^2_T \propto \frac{1}{(T-t)^\alpha}$ / Same for $\hat{\sigma}_T$: YES!

- Use in fact lognormal version of model:

$$d\xi^T_t = \xi^T_t \frac{\lambda}{(T-t)^\alpha} dZ_t$$

- This is called a rough volatility model

- Pricing in a rough volatility model?
SV models - 6

➤ Simulation of the $\xi^T_t$:

\[
d \ln \xi^T_t = \bullet dt + \frac{\lambda}{(T-t)^\alpha} dZ_t
\]

\[
\ln \frac{\xi^T_t}{\xi^T_0} = f(t, T) + \int_0^t \frac{\lambda}{(T-\tau)^\alpha} dZ_\tau
\]

➤ Problem: each $\xi^T_t$ has to be simulated individually: non-Markovian model

⇒ Use discrete 1-month forward variances ⇒ still 120 state variables for a 10-year product

⇒ And this is just for a 1-factor model with all volatilities correlated 100%!

⇒ Rough volatility models not usable in practice

➤ What to do?

➤ In fact no need for the $(T-t)^{-\alpha}$ dependence right down to $(T-t) \to 0/+\infty$

➤ Can we mimic the $(T-t)^{-\alpha}$ form on an interval $[T_{\text{min}}, T_{\text{max}}]$ and get a Markovian (read: usable) model?
SV models - 7

▶ Try exponential form

\[
d \ln \xi_t^T = \bullet dt + e^{-k(T-t)} dB_t
\]

\[
\ln \frac{\xi_t^T}{\xi_0^T} = f(t, T) + \int_0^t e^{-k(T-\tau)} dB_{\tau}
\]

\[
= f(t, T) + e^{-k(T-t)} X_t
\]

where \( X_t = \int_0^t e^{-k(t-\tau)} dB_{\tau} \)

▶ All forward variances are a function of just 1 state variable: \( X_t \)

▶ \( X_t \) is an Ornstein-Ühlenbeck process – easy to simulate

\[
\text{vol}(\hat{\sigma}_T) \propto \frac{1 - e^{-kT}}{kT} \neq \left( \frac{\tau_0}{T} \right)^\alpha
\]

⇒ To reasonably mimic power-law dependence, need at least two factors

⇒ 2-factor model (LB - 2004):

\[
\frac{d \xi_t^T}{\xi_t^T} = (2\nu \alpha_\theta)((1 - \theta)e^{-k_1(T-t)} dW_t^1 + \theta e^{-k_2(T-t)} dW_t^2)
\]

\[
\alpha_\theta = 1/\sqrt{(1 - \theta)^2 + \theta^2 + 2\rho_{12}\theta(1 - \theta)}
\]

+ correlations \( \rho_{1s}, \rho_{2s} \) of \( W^1, W^2 \) with \( S \)
SV models - 8

- Requires simulation of 2 OU processes:

\[ dX_t^1 = -k_1 X_t^1 dt + dW_t^1, \quad dX_t^2 = -k_2 X_t^2 dt + dW_t^2 \]

- For a flat term structure of logswap volatilities – \( \hat{\sigma}_T \) all identical:

\[
\text{vol}(\hat{\sigma}_T) = (\nu \alpha \theta) \sqrt{(1 - \theta)^2 l(k_1 T)^2 + \theta^2 l(k_2 T)^2 + 2 \rho_{12} \theta (1 - \theta) l(k_1 T) l(k_2 T)}
\]

\[
l(x) = \frac{(1 - e^{-x})}{x}
\]

- Example: take \( \tau_0 = 3 \) months, \( \nu_0 = 100\% \), \( \alpha = 0.4 \):

<table>
<thead>
<tr>
<th>Some sets with alpha = 0.4</th>
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<tbody>
<tr>
<td>nu</td>
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<td>theta</td>
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<td>k1</td>
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<td>rho_12</td>
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</table>

- Different sets \( \Rightarrow \) different correlations between the \( \hat{\sigma}_T / \text{ with } S \)
SV models - 9

- Model parameters generate the dynamics of implied volatilities AND the smile
- Can we characterize the relationship between both?

- At order 1 in the volatility of volatility, for a flat term structure of logswap volatilities, the ATM skew for maturity $T$ is given by:

$$ S_T = \frac{1}{2\hat{\sigma}^3_T T} \int_0^T \frac{T-t}{T} E[\langle d\ln S_t, d\hat{\sigma}^2_T, t \rangle] $$

- In a model where:
  - $\hat{\sigma}_T$ is lognormal, with vol($\hat{\sigma}_T$) = $\nu_0 \left( \frac{\tau_0}{T} \right)^\alpha$
  - All $\hat{\sigma}_T$ are 100% correlated
  - Correlation of $S$ and $\hat{\sigma}_T$ is $\rho$

  the ATM skew $S_T$ and curvature $C_T$ are given, resp. at order 1 and 2 in $\nu_0$ by:

$$ S_T = \frac{\rho \nu_0}{2 - \alpha} \left( \tau_0 \right)^\alpha $$

$$ C_T = \frac{\nu_0^2}{\hat{\sigma}_T} \frac{1}{3 - 2\alpha} \left( 1 - 6 \frac{1 - \alpha}{(2 - \alpha)^2} \rho^2 \right) \left( \tau_0 \right)^{2\alpha} $$

*: Note that $C_T$ is not just a function of vol-of-vol*
SV models - conclusion

- Pricing function of SV models:
  \[ P(t, S, \xi, \bullet) \]

- In SV model, we can calibrate a 1-dimensional family of convex payoffs – determines initial curve \( \xi_0 \)
  - For example, logswap or ATM volatilities

- A 2-factor model based on OU processes provides sufficient control on
  - The term-structure of vols of vols
  - Correlation of \( S \) with short- and long-dated implied vols

- Once volatilities and spot/vol correlations are set, smile is set

- How to build a model that takes as input the whole surface of vanilla implied vols \( \hat{\sigma}_{KT} \)?
LSV models

▶ One way: add a local volatility component ⇒ LSV model

\[
\begin{cases}
    dS_t = rS_t \, dt + \sigma(t, S) \sqrt{\xi^T_t} \, dW^S_t \\
    d\xi^T_t = (2\nu_\alpha \theta) \xi^T_t ((1 - \theta)e^{-k_1(T-t)}dW^1_t + \theta e^{-k_2(T-t)}dW^2_t)
\end{cases}
\]

▶ How do we determine \(\sigma(t, S)\) ?

▶ Imagine we have already determined the LV function of the LV model:

\[dS_t = rS_t \, dt + \sigma_{LV}(t, S) \, dW^S_t\]

▶ LV/ LSV models give rise to same densities for \(S_T, \forall T\)

⇒ Giöngy theorem:

\[
\sigma^2(t, S) \, E[\xi^T_t \mid S, t] = \sigma_{LV}^2(t, S) \Rightarrow \sigma(t, S) = \frac{\sigma_{LV}(t, S)}{\sqrt{E[\xi^T_t \mid S, t]}}
\]

▶ How do we compute the conditional expectation?

⇒ Particle method (J. Guyon/P. Henry-Labordère)

▶ \(\sigma(t, S)\) may not exist – no existence theorem, in contrast to LV
LSV models – conclusion

- In LSV model pricing function is:

\[ P(t, S, \hat{\sigma}_{KT}, \bullet) \]

- Params: control on vols of vols / spot/vol and vol/vol correlations

- But there are limitations

- At order 1 in \( \sigma(t, S) \) and order 1 in \( \nu \), ATM skew is given by:

\[
S_T = \frac{1}{2\hat{\sigma}_T^3 T} \int_0^T \frac{T-t}{T} E[\langle d\ln S_t \, d\hat{\sigma}_T^2, t \rangle]
\]

- Enforces relationship between term structure of market ATM skew and spot/vol covariance

- Still, sufficient control in practice

- Present dynamics of index vols altered by large market positions in autocallables

- Need ability to set quite different levels of spot/vol correlation for short/long maturities. Typically:
  - spot/1-year vol: -80%
  - spot/5-year vol: +40%
  - 1-year vol/5-year vol: 0%
Do we actually need a model?

- Option hedging is a stochastic control problem
- Ex: need to price a 1-year ATM call option on the S&P500
- Assume daily delta rehedging. 250 trading days in 1 year: $t_i$
- Need to determine 250 functions $\Delta_i(S)$

- Determine functions $\Delta_i$ so as to minimize variance of final P&L:
  \[
P\&L = -(S_{t_N} - K)^+ + \sum_i \Delta_i(S_{t_i})(S_{t_{i+1}} - S_{t_i})
  \]

- Machine-learning problem
- Take large historical sample of S&P500 – slice it up in 1-year paths
- Parametrize $\Delta_i(S)$ with $N$ neural networks
- Perform minimization with stochastic gradient
- Once done, we have the delta strategy. The option price $P$ is given by:
  \[
P = -E_{\text{histo}}[P\&L]
  \]

- ex: Hedged Monte-Carlo (M. Potters, J.P. Bouchaud, D. Sestovic)
Do we actually need a model? – 2

- Recently it has been proposed to just do away with models and price derivatives as outlined above
- Can include vanilla options as hedge instruments
- First, need a *lot* of histo. data
- Imagine we get a price $P = 7.8$
  - What kind of bid-offer should we quote: $[7.7, 7.9]$? $[7.5, 8.0]$?
  - Could split the sample in sub-samples and look at distribution of prices
  - Need a really big sample
  - Still question: which feature of the dynamics is it that gives rise to this distribution of prices?

- We know it’s actually volatility that determines price ⇒ bid/offer should be quoted in vol points
- But what if we don’t know this?

- Way to go: generate the sample with a model
  - Dynamics of assets does not need to be risk-neutral
  - Use machine-learning algo to determine hedging strategy
  - Can easily include transaction costs
  - Open challenge
Conclusion

▶ Many open challenges in risk-management of derivatives

▶ Symmetric challenges with P&L explain
  ▶ How do we break it down into vanilla-like / non-vanilla-like P&L?

▶ Interesting times ahead!