Volatility modeling: state of the art & outlook

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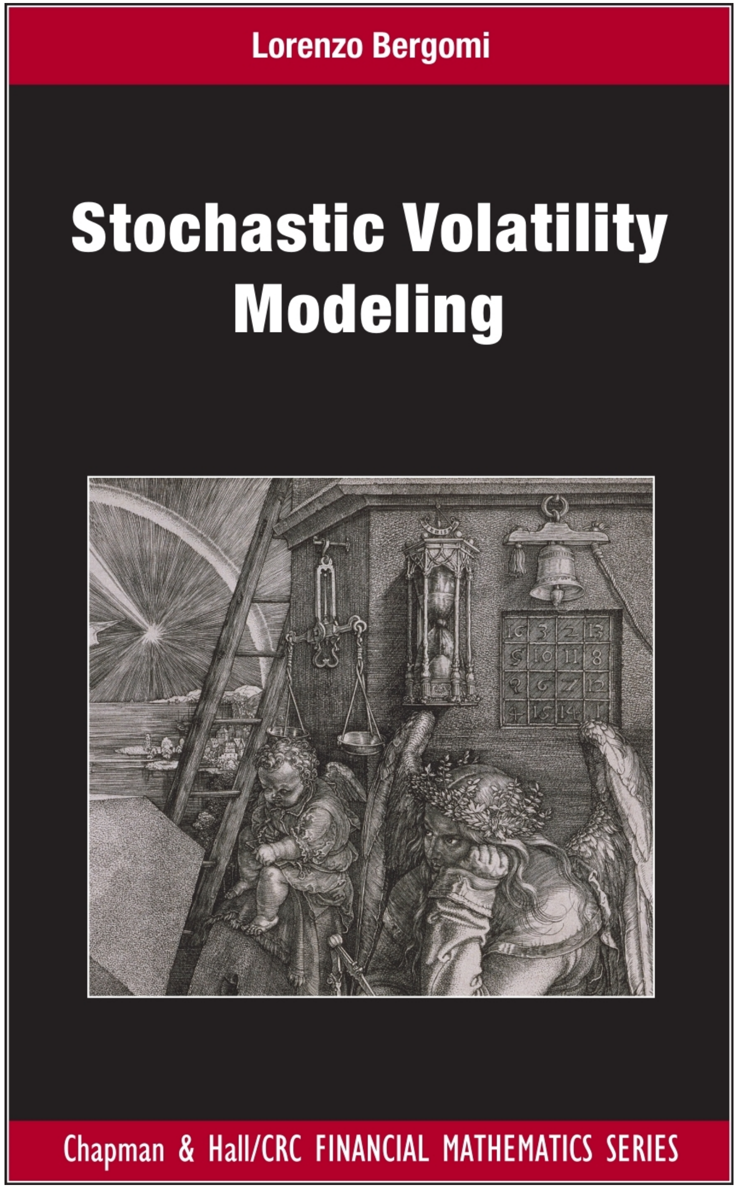
Outline

- The vanilla trader
- The exotic trader
- The local volatility model
- Stochastic volatility models
- Local-stochastic volatility models
- Beyond models?



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Two *ideal types* of derivative traders: the vanilla trader

- Risk-manages vanillas
- Hedge instruments: underlyings/futures/forwards
- Pricing function $P(t, S, \bullet)$. $\bullet \equiv$ model parameters
- P&L of delta-hedged position – assume has sold option + assume zero interest rate

$$P\&L \;=\; -\Big(P(t+\delta t, S+\delta S, \bullet) - P(t, S, \bullet)\Big) + \Delta \delta S$$

- Use Black-Scholes pricing function and take $\Delta \;=\; \frac{dP}{dS}$

$$\frac{dP}{dt} + \frac{1}{2}\hat{\sigma}^2 S^2 \frac{d^2P}{dS^2} \;=\; 0$$

- Expand at order 1 in t, order 2 in δS. P&L reads:

$$P\&L \;=\; \frac{1}{2}S^2\frac{d^2P}{dS^2}\Big(\frac{\delta S^2}{S^2} - \hat{\sigma}^2\delta t\Big)$$

- Notice there are additional contributions to the P&L:

$$P\&L \;=\; \frac{1}{2}S^2\frac{d^2P}{dS^2}\Big(\frac{\delta S^2}{S^2} - \hat{\sigma}^2\delta t\Big) \;+\; \frac{dP}{d\hat{\sigma}}\delta\hat{\sigma} \;+\; \frac{d^2P}{dSd\hat{\sigma}}\delta S\delta\hat{\sigma} \;+\; \frac{1}{2}\frac{d^2P}{d\hat{\sigma}^2}\delta\hat{\sigma}^2 \;+\; \cdots$$

- First portion (Gamma/Theta P&L) is by no means small

Two *ideal types* of derivative traders: the exotic trader

- Risk-manages exotics
- Uses vanillas/delta-hedged vanillas as hedge insruments
- Pricing function takes as input vanilla option prices / vanilla implied volatilities:

$$P(t, S, \hat{\sigma}_{KT}, \bullet) \qquad P(t, S, O_{KT}, \bullet)$$

- Denote by A_i prices of hedging instruments: S and O_{KT}
- P&L of delta and vega-hedged option position – assume zero interest rate

$$P\&L \;=\; -\Big(P(t+\delta t, A+\delta A, \bullet) - P(t, A, \bullet)\Big) + \sum_i \frac{dP}{dA_i}\delta A_i$$

- Expanding at order 1 in δt, order 2 in δA:

$$P\&L \;=\; -\frac{dP}{dt}\delta t \;-\; \frac{1}{2}\sum_{ij}\frac{d^2P}{dA_i dA_j}\delta A_i \delta A_j$$

⇨ How to tell if $P(t, A, \bullet)$ is a non-nonsensical pricing function?

  - There exists a positive matrix $\hat{C}$ such that $\frac{dP}{dt} \;=\; \frac{1}{2}\sum_{ij} A_i A_j \frac{d^2P}{dA_i dA_j}\hat{C}_{ij}$
  - P&L reads:

$$P\&L \;=\; -\frac{1}{2}\Sigma A_i A_j \frac{d^2P}{dA_i dA_j}\left(\frac{\delta A_i}{A_i}\frac{\delta A_j}{A_j} - \hat{C}_{ij}\delta t\right)$$

Two *ideal types* of derivative traders: the exotic trader – 2

- The $\hat{C}_{ij}$ are called break-even covariances
- For a given model, they are functions of $(t, A_i, \bullet)$ with $\bullet \equiv$ model parameters
- They can equivalently be expressed in terms of volatilities and correlations
- In what follows, use implied volatilities $\hat{\sigma}_{KT}$ rather than vanilla option prices
- Ideally we would like to be able to specify breakeven levels:
  - Volatilities of $\hat{\sigma}_{KT}$
  - Correlations of the $\hat{\sigma}_{KT}$ among themselves and with S
- Let us now assess different types of models
  - The Local Volatility (LV) model
  - Stochastic volatility (SV) models
  - Local-stochastic volatility (LSV) models
  - But wait – do we actually need a model?

The LV model

- Historically, LV model introduced as extension of BS with deterministic inst. vol.

$$dS_t = (r-q)S_t dt + \sigma(t, S_t) S_t dW_t$$

- $\sigma(t, S)$ backed out of call prices O_{KT} by the Dupire formula

$$\sigma(T,K)^2 = 2\frac{\frac{dO}{dT} + qO + (r-q)\,K\frac{dO}{dK}}{K^2\frac{d^2O}{dK^2}}$$

- LV function is not a fundamental object, only has an ancillary function
- Pricing function in LV model – that trader uses – is:

$$P(t, S, \hat{\sigma}_{KT}) \quad \text{or} \quad P(t, S, O_{KT})$$

⇨ No parameter ??

- In LV model *for a fixed LV function* all assets are *functions* of t, S

⇨ 1-dimensional Markov representation in terms of t, S, for all assets

- *With a fixed LV function*, pricing function $P^{LV}(t, S, \sigma)$

$$\hat{\sigma}_{KT} \equiv \Sigma^{\mathrm{LV}}_{KT}(t, S, \sigma)$$

The LV model – 2

- Full P&L of naked exotic option position reads - now with interest rates:

$$
\begin{aligned}
P\&L \;=\; & -\frac{dP}{dS}\,(\delta S - rS\delta t) \;-\; \frac{dP}{d\hat{\sigma}_{KT}} \star (\delta\hat{\sigma}_{KT} - \mu_{KT}\delta t) \\
& -\frac{1}{2}S^2\frac{d^2P}{dS^2}\left[\frac{\delta S^2}{S^2} - \sigma^2(t,S)\,\delta t\right] \\
& -\frac{d^2P}{dS d\hat{\sigma}_{KT}} \star S\hat{\sigma}_{KT}\left[\frac{\delta S}{S}\frac{\delta\hat{\sigma}_{KT}}{\hat{\sigma}_{KT}} - \sigma(t,S)\,\nu_{KT}\delta t\right] \\
& -\frac{1}{2}\frac{d^2P}{d\hat{\sigma}_{KT}d\hat{\sigma}_{K'T'}} \star \hat{\sigma}_{KT}\hat{\sigma}_{K'T'}\left[\frac{\delta\hat{\sigma}_{KT}}{\hat{\sigma}_{KT}}\frac{\delta\hat{\sigma}_{K'T'}}{\hat{\sigma}_{K'T'}} - \nu_{KT}\nu_{K'T'}\delta t\right]
\end{aligned}
$$

- where the (lognormal) vol ν_{KT} of $\hat{\sigma}_{KT}$ is given by:

$$
\nu_{KT} \;=\; \frac{1}{\Sigma^{\text{LV}}_{KT}}\frac{d\Sigma^{\text{LV}}_{KT}}{dS}S\sigma(t,S)
$$

- and μ_{KT} is given by:

$$
\mu_{KT} \;=\; \frac{d\Sigma^{\text{LV}}_{KT}}{dt} + \frac{1}{2}\sigma^2(t,S)\,S^2\frac{d^2\Sigma^{\text{LV}}_{KT}}{dS^2} + rS\frac{d\Sigma^{\text{LV}}_{KT}}{dS}
$$

- For a perfectly delta-hedged/vega-hedged position
  - ⇨ The first 2 contributions vanish
  - ⇨ Remaining P&L is of the non-nonsensical type in slide 3

The LV model – 3

- Pricing function takes as input t, the prices of hedge instruments – and that's it
- Not possible to choose the vol of $\hat{\sigma}_{KT}$
  - ν_{KT} is determined by derivative of Σ_{KT}^{LV} with respect to S
  - Σ_{KT}^{LV} determined by local vol function $\sigma(t, S)$
  - Set by the market smile used for calibration

⇨ Correlations between implied vols $= \pm 100\%$

⇨ Correlations between S and $\hat{\sigma}_{KT}$ $= \pm 100\%$

- Can we size up ν_{KT}?

The LV model – 4

- Let us focus on the vol of the ATM vol $\hat{\sigma}_{K=S,T}$, and consider the vol $\tilde{\nu}_{K=S,T}$. It is the volatility of the floating ATM volatility:

$$\tilde{\nu}_{K=S,T} = \frac{1}{\Sigma^{\mathrm{LV}}_{S,T}} \frac{d\Sigma^{\mathrm{LV}}_{K=S,T}}{dS} S\, \sigma(t,S) = \frac{1}{\Sigma^{\mathrm{LV}}_{S,T}} \frac{d\Sigma^{\mathrm{LV}}_{K=S,T}}{d\ln S}\, \sigma(t,S)$$

- For a flat smile, $\sigma(t,S)$ does not depend on S ⇨ neither does $\Sigma^{\mathrm{LV}}_{KT}$

$$⇨ \quad \tilde{\nu}_{K=S,T} = 0$$

- Denote by $\mathcal{S}_\tau$ the ATM skew for maturity τ:

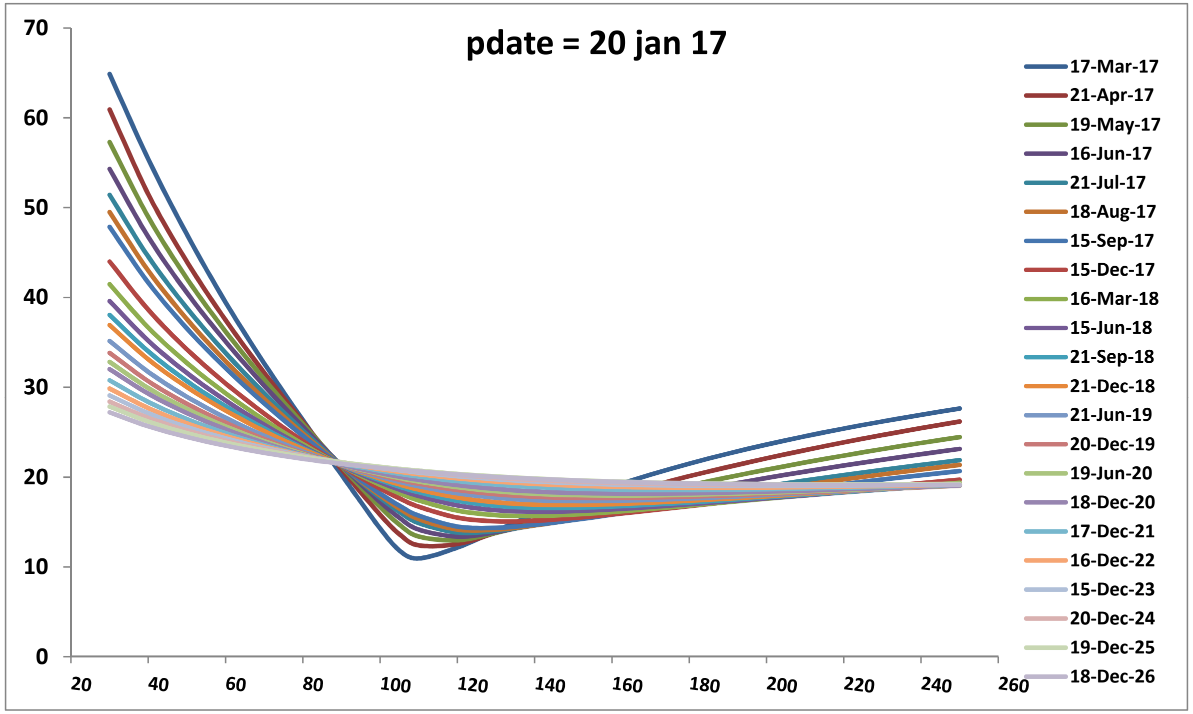
$$\mathcal{S}_\tau = \left.\frac{d\hat{\sigma}_{K\tau}}{d\ln K}\right|_{K=S}$$

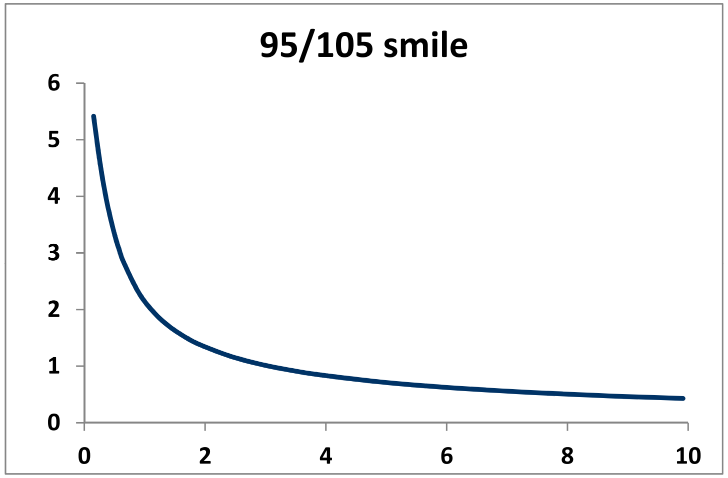
- At order 1 in $\mathcal{S}$:

$$\frac{d\Sigma^{\mathrm{LV}}_{ST}}{d\ln S} = \mathcal{S}_T + \frac{1}{T}\int_0^T \mathcal{S}_\tau\, d\tau$$

The LV model – 5

- Example of index smile. Right: $(\hat{\sigma}_{0.95S,T} - \hat{\sigma}_{1.05S,T})$ as a function of T (years)





- In the LV mode the pricing function of an exotic option reads:

$$P(t, S, \hat{\sigma}_{KT}) \quad \text{or} \quad P(t, S, O_{KT})$$

- No additional parameter to input
- Vols of vols, correlations, are set by market smile
  - Like them ➪ use model
  - Don't like them ➪ don't use model

SV models - 1

- We would like to have a handle on the dynamics of implied vols
  - Not just let model decide – as in the LV model

- Can we model the dynamics of vanilla implied volatilities directly?

$$\begin{cases} dS_t &= rS_t dt + \sigma S_t dW_t^S \\ d\hat{\sigma}_t^{KT} &= \star dt + \bullet \, dW_t^{KT} \end{cases}$$

⇨ Short answer: no

⇨ Implied volatilities of vanilla options are too complicated objects

- Other (convex) European payoffs with simpler implied volatilities?

- Yes: the logswap

$$f(S) = -2 \ln S$$

- In BS, zero interest rate for simplicity, price is given by:

$$P_T(t, S) = -2 \ln S + \frac{1}{2} \hat{\sigma}_T^2 (T - t)$$

⇨ Curve of the $\hat{\sigma}_T$

SV models - 2

- Consider an increasing sequence of dates T_i
- Building block:
  - $+1$ logswap of maturity T_{i+1}
  - -1 logswap of maturity T_i

  ⇨ payoff $= \ln\left(\frac{S_{T_{i+1}}}{S_{T_i}}\right)$
- Price in BS:

$$\begin{aligned} P_{i,i+1}(t,S) &= \frac{1}{2}\hat{\sigma}^2_{i,i+1}\left(T_{i+1}-T_i\right) \\ &= P_{T_{i+1}}(t,S) - P_{T_i}(t,S) \end{aligned}$$

- $\hat{\sigma}^2_{i,i+1}$ is proportional to P, a price ⇨ $\hat{\sigma}^2_{i,i+1}$ has no drift
- Also holds with non-zero interest rates
- Connection with the $\hat{\sigma}_T$:

$$\hat{\sigma}^2_{i,i+1} = \frac{\hat{\sigma}^2_{T_{i+1}}(T_{i+1}-t) - \hat{\sigma}^2_{T_i}(T_i-t)}{T_{i+1}-T_i}$$

- Replace finite difference by derivative ⇨ continuous forward variances ξ^T

$$\xi^T = \frac{d}{d\tau}\left(\hat{\sigma}^2_\tau(\tau-t)\right)\Big|_{\tau=T}$$

SV models - 3

- General form of a stochastic volatility model:

$$\begin{cases} dS_t &= rS_t dt + \sqrt{\xi_t^t}\, dW_t^S \\ d\xi_t^T &= \bullet\, dW_t^T \end{cases}$$

  with correlations $\rho_{ST} dt, \rho_{TT'}$

- This is called a forward-variance model
- All models written on the inst. variance V_t can be written this way

- Example: Heston model – no interest rates:

$$\begin{cases} dS_t &= \sqrt{V_t}\, S_t dW_t \\ dV_t &= -k(V_t - V^0)dt + \nu\sqrt{V_t}\, dZ_t \end{cases} \quad \Rightarrow \quad \begin{cases} dS_t &= \sqrt{\xi_t}\, S_t dW_t \\ d\xi_t^T &= \nu e^{-k(T-t)}\sqrt{\xi_t^t}\, dZ_t \end{cases}$$

- Are the ξ^T easier to model than the $\hat{\sigma}_{KT}$? Yes:
  - they are driftless
  - just ensure $\xi^T \geq 0$

- ξ^T: 1-dimensional set of instruments

⇨ Can calibrate at most one implied vol per maturity

- What should we choose for $\bullet$?

SV models - 4

- Empirically, ATM vols & logswap implied vols are $\approx$ lognormal
- Typically, the inst. (lognormal) volatility of $\hat{\sigma}_T$ has a power-law dependence on T

$$\text{vol}(\hat{\sigma}_T) = \nu_0 \left(\frac{T_0}{T} \right)^{\alpha}$$

- Typical realized values – for indices: $\nu_{3m} \approx 60\%$, $\alpha \in [0.3, 0.6]$
- Similar levels for volatilities of ATM volatilities

- How do achieve this in a forward variance model?

⇨ Receipe

  - Use same Brownian motion for all of the ξ^T
  - Make vol of ξ^T proportional to $(T - t)^{-\alpha}$

$$d\xi_t^T = \frac{\lambda}{(T-t)^{\alpha}} dZ_t$$

SV models - 5

- Let's check. By definition of ξ_t^T:

$$\hat{\sigma}_T^2 = \frac{1}{T-t}\int_t^T \xi_t^\tau \, d\tau$$

- We have:

$$\begin{aligned} d(\hat{\sigma}_T^2) &= \bullet dt + \frac{1}{T-t}\int_t^T d\xi_t^\tau \, d\tau \\ &= \bullet dt + \frac{1}{T-t}\int_t^T \frac{\lambda}{(\tau-t)^\alpha} dZ_t \; d\tau \\ &= \bullet dt + \frac{\lambda}{1-\alpha} \, \frac{1}{(T-t)^\alpha} dZ_t \end{aligned}$$

⇨ Instantaneous vol of $\hat{\sigma}_T^2 \propto \frac{1}{(T-t)^\alpha}$ / Same for $\hat{\sigma}_T$: YES!

- Use in fact lognormal version of model:

$$d\xi_t^T = \xi_t^T \frac{\lambda}{(T-t)^\alpha} dZ_t$$

- This is called a *rough volatility* model
- Pricing in a rough volatility model?

SV models - 6

- Simulation of the ξ^T:

$$
\begin{aligned}
d\ln\xi_t^T &= \bullet dt + \frac{\lambda}{(T-t)^\alpha} dZ_t \\
\ln\frac{\xi_t^T}{\xi_0^T} &= f(t,T) + \int_0^t \frac{\lambda}{(T-\tau)^\alpha} dZ_\tau
\end{aligned}
$$

- Problem: each ξ_t^T has to be simulated individually: non-Markovian model

⇨ Use discrete 1-month forward variances ⇨ still 120 state variables for a 10-year product

⇨ And this is just for a 1-factor model with all volatilities correlated 100%!

⇨ Rough volatility models not usable in practice

- What to do?
- In fact no need for the $(T-t)^{-\alpha}$ dependence right down to $(T-t) \rightarrow 0/+\infty$
- Can we mimic the $(T-t)^{-\alpha}$ form on an interval $[T_{\min}, T_{\max}]$ and get a Markovian (read: usable) model?

SV models - 7

- Try exponential form

$$\begin{aligned} d\ln\xi_t^T &= \bullet dt + \; e^{-k(T-t)} dB_t \\ \ln\frac{\xi_t^T}{\xi_0^T} &= f(t,T) + \int_0^t e^{-k(T-\tau)} dB_\tau \\ &= f(t,T) + e^{-k(T-t)} X_t \end{aligned}$$

where $X_t = \int_0^t e^{-k(t-\tau)} dB_\tau$

- All forward variances are a function of just 1 state variable: X_t
- X_t is an Ornstein-Ühlenbeck process – easy to simulate

$$\text{vol}(\hat{\sigma}_T) \propto \frac{1-e^{-kT}}{kT} \neq \left(\frac{\tau_0}{T}\right)^\alpha$$

⇨ To reasonably mimic power-law dependence, need at least two factors

⇨ 2-factor model (LB - 2004):

$$\begin{aligned} \frac{d\xi_t^T}{\xi_t^T} &= (2\nu\alpha_\theta)\left((1-\theta)e^{-k_1(T-t)} dW_t^1 + \theta e^{-k_2(T-t)} dW_t^2\right) \\ \alpha_\theta &= 1/\sqrt{(1-\theta)^2 + \theta^2 + 2\rho_{12}\theta\,(1-\theta)} \end{aligned}$$

$+$ correlations ρ_{1S}, ρ_{2S} of W^1, W^2 with S

SV models - 8

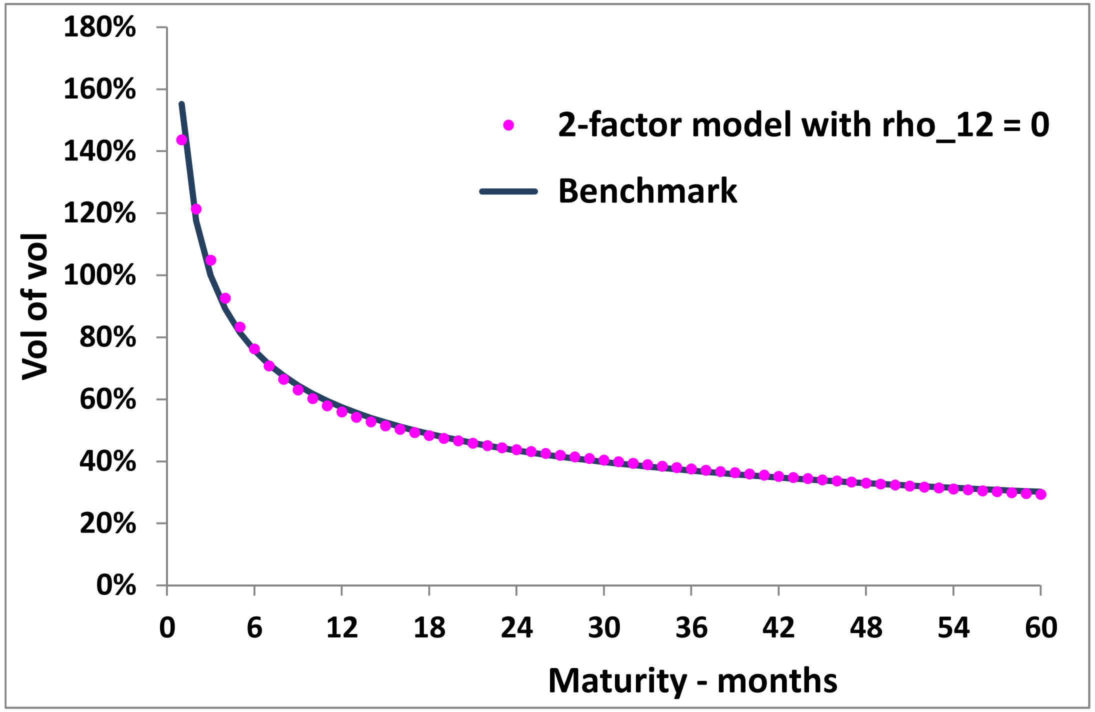
- Requires simulation of 2 OU processes:

$$dX_t^1 = -k_1 X_t^1 dt + dW_t^1, \qquad dX_t^2 = -k_2 X_t^2 dt + dW_t^2$$

- For a flat term structure of logswap volatilities – $\hat{\sigma}_T$ all identical:

$$\begin{aligned} \text{vol}\left(\widehat{\sigma}_T\right) &= \left(\nu\alpha_\theta\right)\sqrt{(1-\theta)^2 I(k_1 T)^2 + \theta^2 I(k_2 T)^2 + 2\rho_{12}\theta\,(1-\theta) I(k_1 T) I(k_2 T)} \\ I(x) &= \left(1-e^{-x}\right)/x \end{aligned}$$

- Example: take $\tau_0 = 3\text{months}$, $\nu_0 = 100\%$, $\alpha = 0.4$:



Some sets with alpha = 0.4		
nu	120.9%	135.8%
theta	57.9%	30.1%
k1	0.58	2.59
k2	1.19	0.32
rho_12	-95%	-50%
nu	174.0%	178.2%
theta	24.5%	23.8%
k1	5.35	6.02
k2	0.28	0.27
rho_12	0%	20%
nu	185.1%	190.1%
theta	23.1%	22.8%
k1	7.26	8.34
k2	0.24	0.22
rho_12	60%	99%

- Different sets ⇨ different correlations between the $\hat{\sigma}_T$ / with S

SV models - 9

- Model parameters generate the dynamics of implied volatilities AND the smile
- Can we characterize the relationship between both?

- At order 1 in the volatility of volatility, for a flat term structure of logswap volatilities, the ATM skew for maturity T is given by:

$$\mathcal{S}_T = \frac{1}{2\hat{\sigma}_T^3 T} \int_0^T \frac{T-t}{T} E[\langle d\ln S_t \ d\hat{\sigma}^2_{T,t} \rangle]$$

- In a model where:
  - $\hat{\sigma}_T$ is lognormal, with $\text{vol}(\hat{\sigma}_T) = \nu_0 \left(\frac{\tau_0}{T}\right)^{\alpha}$
  - All $\hat{\sigma}_T$ are 100% correlated
  - Correlation of S and $\hat{\sigma}_T$ is ρ

  the ATM skew $\mathcal{S}_T$ and curvature $\mathcal{C}_T$ are given, resp. at order 1 and 2 in ν_0 by:

$$\mathcal{S}_T = \frac{\rho\nu_0}{2-\alpha}\left(\frac{\tau_0}{T}\right)^{\alpha}$$

$$\mathcal{C}_T = \frac{\nu_0^2}{\hat{\sigma}_T} \frac{1}{3-2\alpha}\left(1 - 6\frac{1-\alpha}{(2-\alpha)^2}\rho^2\right)\left(\frac{\tau_0}{T}\right)^{2\alpha}$$

⇨ Note that $\mathcal{C}_T$ is not just a function of vol-of-vol

SV models - conclusion

- Pricing function of SV models:

$$P(t, S, \xi, \bullet)$$

- In SV model, we can calibrate a 1-dimensional family of convex payoffs – determines initial curve ξ_0
  - For example, logswap or ATM volatilities
- A 2-factor model based on OU processes provides sufficient control on
  - The term-structure of vols of vols
  - Correlation of S with short- and long-dated implied vols
- Once volatilities and spot/vol correlations are set, smile is set
- How to build a model that takes as input the whole surface of vanilla implied vols $\hat{\sigma}_{KT}$?

LSV models

- One way: add a local volatility component ⇨ LSV model

$$\begin{cases} dS_t &= rS_t dt + \sigma(t,S)\ \sqrt{\xi_t^t}\ dW_t^S \\ d\xi_t^T &= (2\nu\alpha_\theta)\ \xi_t^T\ \left((1-\theta)e^{-k_1(T-t)}dW_t^1 + \theta e^{-k_2(T-t)}dW_t^2\right) \end{cases}$$

- How do we determine $\sigma(t,S)$?

- Imagine we have already determined the LV function of the LV model:

$$dS_t \;=\; rS_t dt + \sigma_{LV}(t,S)\ dW_t^S$$

- LV/ LSV models give rise to same densities for $S_T, \forall T$

⇨ Giöngy theorem:

$$\sigma^2(t,S)\ E[\xi_t^t \mid S,t] \;=\; \sigma_{LV}^2(t,S) \qquad ⇨ \qquad \sigma(t,S) \;=\; \frac{\sigma_{LV}(t,S)}{\sqrt{E[\xi_t^t \mid S,t]}}$$

- How do we compute the conditional expectation?

⇨ Particle method (J. Guyon/P. Henry-Labordère)

- $\sigma(t,S)$ may not exist – no existence theorem, in contrast to LV

LSV models – conclusion

- In LSV model pricin function is:

$$P(t, S, \hat{\sigma}_{KT}, \bullet)$$

- Params: control on vols of vols / spot/vol and vol/vol correlations
- But there are limitations
- At order 1 in $\sigma(t, S)$ and order 1 in ν, ATM skew is given by:

$$\mathcal{S}_T = \frac{1}{2\hat{\sigma}_T^3 T} \int_0^T \frac{T-t}{T} E[\langle d\ln S_t \ d\hat{\sigma}_{T,t}^2 \rangle]$$

⇨ Enforces relationship between term structure of market ATM skew and spot/vol covariance

- Still, sufficient control in practice
- Present dynamics of index vols altered by large market positions in autocallables
- Need ability to set quite different levels of spot/vol correlation for short/long maturities. Typically:
  - spot/1-year vol: -80%
  - spot/5-year vol: +40%
  - 1-year vol/5-year vol: 0%

Do we actually need a model ?

- Option hedging is a stochastic control problem
- Ex: need to price a 1-year ATM call option on the S&P500
- Assume daily delta rehedging. 250 trading days in 1 year: t_i
- Need to determine 250 *functions* $\Delta_i(S)$
- Determine functions Δ_i so as to minimize variance of final P&L:

$$P\&L \;=\; -(S_{t_N} - K)^+ \;+\; \sum_i \Delta_i(S_{t_i})(S_{t_{i+1}} - S_{t_i})$$

⇨ Machine-learning problem
⇨ Take large historical sample of S&P500 – slice it up in 1-year paths
⇨ Parametrize $\Delta_i(S)$ with N neural networks
⇨ Perform minimization with stochastic gradient
⇨ Once done, we have the delta strategy. The option price P is given by:

$$P \;=\; -E_{\text{histo}}[P\&L]$$

- ex: Hedged Monte-Carlo (M. Potters, J.P. Bouchaud, D. Sestovic)

Do we actually need a model ? – 2

- Recently it has been proposed to just do away with models and price derivatives as outlined above
- Can include vanilla options as hedge instruments
- First, need a *lot* of histo. data
- Imagine we get a price $P = 7.8$
  - What kind of bid-offer should we quote: $[7.7, 7.9]$? $[7.5, 8.0]$?
  - Could split the sample in sub-samples and look at distribution of prices
  - Need a really big sample
  - Still question: which feature of the dynamics is it that gives rise to this distribution of prices?

⇨ We know it's actually volatility that determines price ⇨ bid/offer should be quoted in vol points

- But what if we don't know this?
- Way to go: generate the sample with a model
  - Dynamics of assets does not need to be risk-neutral
  - Use machine-learning algo to determine hedging strategy
  - Can easily include transaction costs
  - Open challenge

Conclusion

- Many open challenges in risk-management of derivatives
- Symmetric challenges with P&L explain
  - How do we break it down into vanilla-like / non-vanilla-like P&L ?
- Interesting times ahead!